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## A THEORY OF SIMPLICITY IN GAMES AND MECHANISM DESIGN

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We study extensive-form games and mechanisms allowing agents that plan for only a subset of future decisions they may be called to make (the *planning horizon*). Agents may update their so-called *strategic plan* as the game progresses and new decision points enter their planning horizon. We introduce a family of *simplicity* standards which require that the prescribed action leads to unambiguously better outcomes, no matter what happens outside the planning horizon. We employ these standards to explore the trade-off between simplicity and other objectives, to characterize simple mechanisms in a wide range of economic environments, and to delineate the simplicity of common mechanisms such as posted prices and ascending auctions, with the former being simpler than the latter.

KEYWORDS: One-step simplicity, (strong) obvious strategy-proofness, planning horizon, limited foresight, price and priority mechanisms, ascending auctions, extensive-form games.

#### 1. INTRODUCTION

CONSIDER A GROUP OF AGENTS who must come together to make a choice from some set of potential outcomes that will affect each of them. This can be modeled as an extensive-form game, with the final outcome determined by the decisions made by the agents during the game. To ensure that the final outcome satisfies desirable normative properties (e.g.,

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efficiency or revenue maximization), the standard approach in mechanism design is to provide agents with incentives to play in a predictable and desirable way. For instance, the designer may use a Bayesian or dominant-strategy incentive-compatible direct mechanism where it is in each agent's best interest to simply report all of their private information truthfully. This approach succeeds so long as the participants understand that being truthful is in their interest (for instance, if the designer has the ability to successfully teach the agents how to play). However, there is accumulating empirical evidence of agents not reporting the truth in such mechanisms.<sup>1</sup> Bayesian or dominant-strategy mechanisms are thus sometimes not sufficiently simple for participants to play as expected. Using simpler mechanisms can reduce such strategic confusion. It may also lower participation costs, attract participants, and equalize opportunities across participants with different levels of access to information and strategic sophistication. Additionally, designing simpler mechanisms requires less information about participants' beliefs.<sup>2</sup>

What mechanisms, then, are actually simple to play? We address this question by introducing a general class of simplicity standards for games and mechanism design. We use these standards to assess the restrictions that simplicity imposes on the mechanism designer and to characterize simple mechanisms for a broad range of social choice environments with and without transfers.<sup>3</sup> Analogously to the revelation principle for Bayesian mechanism design, we construct classes of mechanisms that limit the space over which a designer interested in implementing a simple mechanism must search.<sup>4</sup> As applications, we provide simplicity-based microfoundations for popular mechanisms such as posted prices, priority mechanisms, and ascending auctions.

The main innovation in our approach is a departure from the standard assumption that agents have unlimited foresight and are able to plan a complete strategy for every possible future contingency. Rather, we allow for agents with limited foresight who, each time they are called to play, make plans for only a subset of possible future moves, their current planning horizon. We refer to these plans as partial strategic plans.<sup>5</sup> A partial strategic plan is simply dominant if the called-for action is weakly better than any alternative, irrespective of what happens at decision points not planned for. As the game progresses, agents may update their strategic plans, and choose an action that is different from what they planned in the past. This potential for updating is what differentiates strategic plans from the standard game-theoretic concept of a strategy.<sup>6</sup>

<sup>&</sup>lt;sup>1</sup>See, for example, Kagel, Harstad, and Levin (1987), Li (2017b), Hassidim, Romm, and Shorrer (2016), Rees-Jones (2017, 2018), Shorrer and Sóvágó (2018), and Artemov, Che, and He (2017).

<sup>&</sup>lt;sup>2</sup>See Vickrey (1961) for participation costs, Spenner and Freeman (2012) for attracting participants, Pathak and Sönmez (2008) for leveling the playing field, and Wilson (1987) and Bergemann and Morris (2005) for a designer's informational requirements.

<sup>&</sup>lt;sup>3</sup>Examples include auctions (Vickrey (1961), Riley and Samuelson (1981), Myerson (1981)), voting (Arrow (1963)), school choice (Abdulkadiroğlu and Sönmez (2003)), organ exchange (Roth, Sönmez, and Ünver (2004)), course allocation (Sönmez and Ünver (2010), Budish and Cantillon (2012)), and refugee resettlement (Jones and Teytelboym (2016), Delacrétaz, Kominers, and Teytelboym (2016)).

<sup>&</sup>lt;sup>4</sup>Direct mechanisms are not necessarily simple, and hence the revelation principle does not extend to simple extensive-form games; cf. Li (2017b).

<sup>&</sup>lt;sup>5</sup>Savage (1954) wrestled with whether decision-makers should be modeled as "look before you leap" (create a complete contingent plan for all possible future decisions one may face) or "you can cross that bridge when you come to it" (make choices as they arise). While standard strategic concepts of game theory formalize the former modeling option, our approach formalizes the latter.

<sup>&</sup>lt;sup>6</sup>We are agnostic as to whether the agents are sophisticated and understand that their plans might be updated, or whether the agents are naive about this possibility. The future actions in the partial strategic plan merely ensure the optimality of the initial action.

We model variations in simplicity and required foresight ability by varying agents' planning horizons. This gives rise to a family of simple dominance standards. The stronger the simplicity standard—that is, the fewer information sets in the current planning horizon—the more robust the corresponding mechanism is to agents who can plan for only limited future horizons. We show that the longer the planning horizon of the agents, the more social choice rules a designer can implement in a simply dominant way. Furthermore, any implementable social rule can be implemented via a perfect-information extensive-form game.

We focus on special cases of simple dominance in which agents are able to plan some exogenously given number  $k \in \{0, 1, ..., \infty\}$  of future moves and analyze three such special cases of simple dominance in detail:

- $k = \infty$ : each agent's planning horizon consists of all information sets at which this agent moves (and contains no other information sets); in other words, at each information set, an agent can plan the actions they will take at any future information set at which they may be called to play. In this case, simple dominance becomes equivalent to Li's (2017b) obvious dominance, and so we refer to the resulting simply dominant strategic plans as *obviously dominant*, and the corresponding mechanisms as *obviously strategy-proof (OSP)*.
- k = 1: each agent's planning horizon consists of their current information set and only the first information sets at which they may be called to play in the continuation game; in other words, agents are able to plan at most one move ahead at a time. We refer to the resulting simply dominant strategic plans as *one-step dominant*, and the corresponding mechanisms as *one-step simple (OSS)*.
- k = 0: each agent's planning horizon consists only of their current information set; in other words, agents cannot plan for any moves in the future. We refer to the resulting simply dominant strategic plans as *strongly obviously dominant*, and the corresponding mechanisms as *strongly obviously strategy-proof (SOSP)*.

The above standards are nested: strongly obviously dominant strategic plans are one-step dominant, which in turn are obviously dominant. Obvious dominance is the most permissive of these standards. It relies on the assumption that agents can create a complete plan for all possible contingencies going forward, and further are able to perform backward induction over at least their own future actions (though not over the actions of their opponents). For instance, consider the game of chess: assuming that White can always force a win, any winning strategy of White is obviously dominant; yet, the strategic choices in chess are far from obvious. Winning strategies in chess require looking many steps into the future, and thus are not one-step dominant nor strongly obviously dominant. Games that admit one-step and/or strongly obviously dominant strategies do not require agents to have such lengthy foresight.

For the above three simplicity standards we ask: which mechanisms are simple? For obvious dominance, we focus on social choice environments without transfers, hence complementing Li (2017b), who focused on the case with transfers. We show that OSP games can be represented as *millipede games*. In a millipede game, each time an agent is called to move, she is presented with some subset of payoff-equivalent outcomes, or more simply *payoffs*, that she can 'clinch'. Clinching corresponds to receiving a payoff for sure, and leaving the game. The agent may also be given the opportunity to 'pass'. If the agent

<sup>&</sup>lt;sup>7</sup>We show that a strategic plan is simply dominant if and only if in every game an agent may confuse with the actual game being played, the strategic plan is weakly dominant in the standard sense (Theorem 3). Li (2017b) provided a related behavioral microfoundation for his obvious dominance, on which we build.

passes, she remains in the game, with the potential of being offered better clinching options in the future. Agents are sequentially presented with such clinch-or-pass choices until all agents' payoffs are determined. The millipede class includes as special cases some familiar, and intuitively simple, games, such as serial dictatorships. However, the millipede class also admits other games that are rarely observed in market-design practice, and whose strategy-proofness is not necessarily immediately clear. In particular, similarly to chess, some millipede games require agents to look far into the future and to perform potentially complicated backward induction reasoning (see Figure 2 in Section 4.2 for an example).

We next study one-step dominance in environments both with and without transfers. We show that in the binary allocation environments with transfers studied by Li (2017b)—which encompass canonical special cases such as single-unit auctions and binary public good choice—any one-step simple mechanism is equivalent to a personal clock auction. This strengthens Li's result that OSP mechanisms are equivalent to personal clock auctions. In particular, any social choice rule that is implementable in obviously dominant strategies is also implementable in one-step dominant strategic plans. In no-transfer environments, one-step simplicity eliminates the complex OSP millipede games discussed above (and also eliminates games such as chess). Indeed, we characterize OSS millipede games as those that satisfy a property we call monotonicity. Monotonicity ensures that, at the first time an agent may be called to play again in the continuation game that begins following any of her current actions, the agent's best clinching option will be weakly better than anything she could have clinched in the past. Monotonic games seem particularly simple, since the agent only needs to recognize that she can do no worse at her very next move if she remains in the game.<sup>8</sup>

For strong obvious dominance, we show that SOSP games do not require agents to look far into the future and perform lengthy backward induction: in all such games, each agent has essentially at most one payoff-relevant move. Strongly obviously dominant strategic plans are incentive-compatible even for agents concerned about trembles, or who have time-inconsistent preferences. Building on this insight, we show that all SOSP mechanisms can be implemented as *sequential choice mechanisms* in which each agent moves at most once, and, at this move, is offered a choice from a menu of options. If the menu has three or more options, then the agent's final payoff is what they choose from the menu. If the menu has only two options, then the agent's final payoff might depend on other agents' choices, but truthfully indicating the preferred option is the strongly obviously dominant choice. The offered menu may include prices, in which case we call the mechanism a *sequential posted price mechanism*. The strong obvious dominance of these mechanisms provides an explanation of the popularity of posted prices, a ubiquitous sales procedure.<sup>9</sup>

Our construction of simplicity criteria is inspired by Li (2017b), who formalized obvious strategy-proofness and established its desirability as an incentive property. We go beyond his work in two ways. First, we introduce gradated standards of simplicity, which allow us to assess the trade-off between simplicity and implementation flexibility. Second, we

<sup>&</sup>lt;sup>8</sup>Monotonicity is a generalization of a similar feature of ascending auctions: in an ascending auction, if an agent passes (continues in the auction), at any next move, she is able to drop out (clinch the zero payoff), except if she wins. Since the zero payoff is the only clinchable payoff and this payoff is always clinchable (except if the agent wins), ascending auctions are monotonic. Such monotonicity is also satisfied by Li's personal clock auctions.

<sup>&</sup>lt;sup>9</sup>For earlier microfoundations of posted prices, see Hagerty and Rogerson (1987) and Copic and Ponsati (2016).

provide simplicity-based microfoundations for popular mechanisms such as posted prices and priority mechanisms. Following up on Li's work, but preceding ours, Ashlagi and Gonczarowski (2018) showed that stable mechanisms such as Deferred Acceptance (DA) are not obviously strategy-proof, except in very restrictive environments where DA simplifies to an obviously strategy-proof game with a 'clinch-or-pass' structure similar to simple millipede games (though they did not describe it in these terms). Other related papers include Troyan (2019), who studied obviously strategy-proof allocation via the popular Top Trading Cycles (TTC) mechanisms, and provided a characterization of the priority structures under which TTC are OSP-implementable.<sup>10</sup> Following our work, Arribillaga, Massó, and Neme (2020) and (forthcoming) characterized the voting rules that are obviously strategy-proof on domains of single-peaked preferences. Bade and Gonczarowski (2017) studied obviously strategy-proof and efficient social choice rules in several environments. Mackenzie (2020) introduced the notion of a "round table mechanism" for OSP implementation and drew parallels with the standard Myerson-Riley revelation principle for direct mechanisms. There has been less work that goes beyond Li's obvious dominance. Li (2017a) extended his ideas to an ex post equilibrium context, while Zhang and Levin (2017a, 2017b) provided decision-theoretic foundations for obvious dominance and explored weaker incentive concepts. 11

Our work also contributes to the understanding of limited foresight and limits on backward induction. Other work in this area, with different approaches from ours, includes Jehiel (1995, 2001) on limited foresight equilibrium in which players' forecasts are correct, Gabaix, Laibson, Moloche, and Weinberg (2006) on directed cognition, and Ke's (2019) axiomatization of bounded-horizon backward induction. A major difficulty for models of imperfect foresight is how an agent takes into account a future they are unable to foresee; we resolve this difficulty by designing games in which all resolutions of the unforeseen lead the agent to the same current decision. 12 As a by-product, our mechanisms work well when agents are intertemporally inconsistent, for instance because they face Knightian uncertainty or optimize against multiple priors (as, e.g., in Knight (1921) and Bewley (1987)) or have time-inconsistent preferences (as, e.g., in Strotz (1956) and Laibson (1997)). Finally, this paper adds to our understanding of dominant incentives, efficiency, and fairness in settings with and without transfers. In settings with transfers, these questions were studied by, for example, Vickrey (1961), Clarke (1971), Groves (1973), Green and Laffont (1977), Holmstrom (1979), Dasgupta, Hammond, and Maskin (1979), and Hagerty and Rogerson (1987). In settings without transfers, in addition to Gibbard (1973, 1977) and Satterthwaite (1975) and the allocation papers mentioned above, the literature on mechanisms satisfying these key objectives includes Ehlers (2002) and Pycia and Unver (2020, 2017) who characterized efficient and group strategy-proof mechanisms in settings with

<sup>&</sup>lt;sup>10</sup>Li showed that the classic TTC mechanism of Shapley and Scarf (1974), in which each agent starts by owning exactly one object, is not obviously strategy-proof. Following our and Troyan's work, Mandal and Roy (2022) characterized the priority structures under which Hierarchical Exchange of Pápai (2000) and Trading Cycles (group strategy-proof and efficient mechanisms) of Pycia and Ünver (2017) are OSP-implementable; cf. also Mandal and Roy (2022).

<sup>&</sup>lt;sup>11</sup>Also of note is Glazer and Rubinstein (1996), who argued that extensive-form games may simplify the solution of normal-form games, and Loertscher and Marx (2020), who studied environments with transfers and constructed a prior-free obviously strategy-proof mechanism that becomes asymptotically optimal as the number of buyers and sellers grows. A different strategic perspective on simplicity in mechanism design was explored by Börgers and Li (2019).

<sup>&</sup>lt;sup>12</sup>The issue of accounting for the unforeseen is also crucial for the analyses of incomplete contracts (e.g., Maskin and Tirole (1999)) and unawareness (e.g., Karni and Viero (2013)). Agents who rely on incomplete models have been also studied in the context of persuasion (e.g., Schwartzstein and Sunderam (2021)).

single-unit demand, and Pápai (2001) and Hatfield (2009) who provided such characterizations for settings with multi-unit demand.<sup>13</sup> Liu and Pycia (2016), Pycia (2011), Morrill (2015), Hakimov and Kesten (2018), Ehlers and Morrill (2020), and Troyan, Delacretaz, and Kloosterman (2020) characterized mechanisms that satisfy incentive, efficiency, and fairness objectives.

#### 2. MODEL

## 2.1. Preferences

Let  $\mathcal{N} = \{i_1, \dots, i_N\}$  be a set of agents, and  $\mathcal{X}$  a finite set of outcomes.\(^{14} An outcome might involve a monetary transfer. Each agent has a preference ranking over outcomes, where, for  $x, y \in \mathcal{X}$ , we write  $x \succsim_i y$  to denote that x is weakly preferred to y. We allow for indifferences, and write  $x \succsim_i y$  if  $x \succsim_i y$  and  $y \succsim_i x$ . For any  $\succsim_i$ , we let  $\succ_i$  denote the corresponding strict preference relation, that is,  $x \succ_i y$  if  $x \succsim_i y$  but not  $y \succsim_i x$ . We use  $\mathcal{P}_i$  to denote the domain of agent i's preferences, and refer to  $\succsim_i$  as agent i's type.

We allow incomplete information through the standard imperfect-information construction of a meta-game in which Nature moves first and determines agents' types, and only then the designed game/mechanism is played. Due to the nature of the dominance properties we study, we do not need to make any assumptions on agents' beliefs about others' types nor on how agents evaluate lotteries.<sup>15</sup>

#### 2.2. Extensive-Form Games

To determine the outcome, the planner designs a game  $\Gamma$  for the agents to play. We consider imperfect-information, extensive-form games with perfect recall. These are defined in the standard way: there is a finite collection of partially ordered *histories*,  $\mathcal{H}$ . We write  $h' \subseteq h$  to denote that  $h' \in \mathcal{H}$  is a subhistory of  $h \in \mathcal{H}$ , and  $h' \subset h$  when  $h' \subseteq h$  but  $h \neq h'$ . Terminal histories are denoted with bars, that is,  $\bar{h}$ . Each  $\bar{h} \in \mathcal{H}$  is associated with an outcome in  $\mathcal{X}$ . At every non-terminal history  $h \in \mathcal{H}$ , one agent, denoted  $i_h$ , is called to play and chooses an *action* from a finite set A(h). We write h' = (h, a) to denote the history h' that is reached by starting at history h and following the action  $h \in \mathcal{A}(h)$ . To avoid trivialities, we assume that no agent moves twice in a row and that |A(h)| > 1 for all non-terminal  $h \in \mathcal{H}$ . To capture random mechanisms, we also allow for histories h at which a non-strategic agent, Nature, is called to move. When Nature moves, she selects an action from h and h according to some known probability distribution.

The set of histories at which agent *i* moves is denoted  $\mathcal{H}_i = \{h \in \mathcal{H} : i_h = i\}$ . We partition  $\mathcal{H}_i$  into *information sets* and denote this partition by  $\mathcal{I}_i$ . For any information set  $I \in \mathcal{I}_i$ 

<sup>&</sup>lt;sup>13</sup>Pycia and Ünver (2020) characterized individually strategy-proof and Arrovian efficient mechanisms. For an analysis of these issues under additional feasibility constraints, see also Dur and Ünver (2019) and Root and Ahn (2020).

<sup>&</sup>lt;sup>14</sup>Assuming  $\mathcal{X}$  is finite simplifies the exposition and is satisfied in the examples listed in the Introduction. This assumption can be relaxed. For instance, our analysis goes through with no substantive changes if we allow infinite  $\mathcal{X}$  endowed with a topology such that agents' preferences are continuous in this topology and the relevant sets of outcomes are compact.

<sup>&</sup>lt;sup>15</sup>It is natural to assume that an agent weakly prefers lottery  $\mu$  over  $\nu$  whenever for all outcomes  $x \in \operatorname{supp}(\mu)$  and  $y \in \operatorname{supp}(\nu)$  this agent weakly prefers x over y. This mild assumption is satisfied for expected utility agents, as well as for agents who prefer  $\mu$  to  $\nu$  as soon as  $\mu$  first-order stochastically dominates  $\nu$ . While our results do not rely on this assumption, it ensures that dominant actions always lead to weakly preferred lotteries over outcomes.

and  $h,h'\in I$  and any subhistories  $\tilde{h}\subseteq h$  and  $\tilde{h}'\subseteq h'$  at which i moves, at least one of the following two symmetric conditions obtains: either (i) there is a history  $\tilde{h}^*\subseteq \tilde{h}$  such that  $\tilde{h}^*$  and  $\tilde{h}'$  are in the same information set,  $A(\tilde{h}^*)=A(\tilde{h}')$ , and i makes the same move at  $\tilde{h}^*$  and  $\tilde{h}'$ , or (ii) there is a history  $\tilde{h}^*\subseteq \tilde{h}'$  such that  $\tilde{h}^*$  and  $\tilde{h}$  are in the same information set,  $A(\tilde{h}^*)=A(\tilde{h})$ , and i makes the same move at  $\tilde{h}^*$  and  $\tilde{h}$ . We denote by  $I(h)\in\mathcal{I}_i$  the information set containing history h. We say that an information set  $I_1$  precedes information set  $I_2$  if there are  $h_1\in I_1$  and  $h_2\in I_2$  such that  $h_1\subseteq h_2$ . If  $I_1$  precedes  $I_2$ , we write  $I_1\leq I_2$  (and  $I_1< I_2$  if  $I_1\neq I_2$ ); we then also say that  $I_2$  follows  $I_1$  and that  $I_2$  is a continuation of  $I_1$ . An outcome  $I_1$  is a continuation set  $I_2$  if there is  $I_1$  and a terminal history  $I_2$  he such that  $I_3$  obtains at  $I_4$ .

#### 3. SIMPLE DOMINANCE

What extensive-form games are simple to play? Intuitively, choosing from a menu of outcomes—for example, a take-it-or-leave-it opportunity to buy an object at a posted price, or a choice from a set of objects in an extensive-form serial dictatorship (cf. Section 4.3)—entails simpler strategic considerations than those faced by a bidder in an ascending clock auction. Similarly, ascending clock auctions are simpler than complex games such as chess. We propose a class of simplicity standards that allows us to differentiate the strategic simplicity of these and other games.

In defining our class of simplicity standards, we relax the standard assumption of economic analysis that players can analyze and plan their actions arbitrarily far into the future. Such unlimited foresight assumptions are embedded in standard game-theoretic concepts of backward induction, dynamic programming, perfect Bayesian equilibrium, iterated dominance, weak dominance, and Li's obvious dominance. In relaxing the foresight assumption, we build on the pioneering approach of Li (2017b), whose obvious dominance allows for agents who do not reason carefully about what their opponents will do, while still requiring that they search deep into the game with regard to their future self. Li's agents know the structure of precedence among the information sets at which they move and the sets of outcomes that could possibly obtain conditional on any sequence of their own actions (though not conditional on their opponents' actions). For instance, if White has a winning strategy in chess—that is, at the start of the game, White knows what to do at any possible future configuration of the board to ensure a victory—then this strategy is also obviously dominant. We relax Li's foresight assumptions, only maintaining that players know possible outcomes of actions and precedence relations for information sets in their planning horizon (i.e., those information sets for which the agent plans).

The key innovation in our framework is that an agent may update their plan as the game is played. In other words, we allow the agent's perception of the strategic situation, and hence, their planned actions, to vary as the game progresses. To differentiate them from the standard game-theoretic notion of a "strategy" as a complete contingent plan of action, we refer to these objects as "strategic plans," introduced in the next subsection.

## 3.1. Strategic Plans

Formally, each information set  $I^* \in \mathcal{I}_i$  at which agent i moves has an associated set of continuation information sets  $\mathcal{I}_{i,I^*} \subseteq \{I \in \mathcal{I}_i | I \geq I^*\}$  that are *simple from the perspective of*  $I^*$ ; we call  $\mathcal{I}_{i,I^*}$  agent i's *planning horizon* at  $I^*$ . We assume that  $I^* \in \mathcal{I}_{i,I^*}$ , but otherwise,

the only restriction is that  $\mathcal{I}_{i,I^*} \subseteq \mathcal{I}_i$ . <sup>16</sup> A (partial) strategic plan  $S_{i,I^*}(\succ_i)$  for agent i of type  $\succ_i$  at information set  $I^*$  maps each simple information set  $I \in \mathcal{I}_{i,I^*}$  to an action in A(I). <sup>17</sup> Note that a strategic plan does not specify the play at all continuation information sets at which i may be called to move; rather, the strategic plan at  $I^*$  only specifies an action at the information sets in the planning horizon at  $I^*$ . Sets of strategic plans  $(S_{i,I^*}(\succ_i))_{I^*\in\mathcal{I}_i}$  and  $(S_{i,I^*}(\succ_i))_{I^*\in\mathcal{I}_i,\succ_i\in\mathcal{P}_i}$  of agent i are called strategic collections.

An extensive-form mechanism  $(\Gamma, S_{\mathcal{N}, \mathcal{I}})$ , or simply a mechanism, is an extensive-form game  $\Gamma$  together with a profile of strategic collections,  $S_{\mathcal{N}, \mathcal{I}} = ((S_{i,I^*}(\succ_i))_{I^* \in \mathcal{I}_i, \succ_i \in \mathcal{P}_i})_{i \in \mathcal{N}}$ . For any strategic collection  $(S_{i,I^*}(\succ_i))_{I^* \in \mathcal{I}_i}$ , we define the induced strategy  $\hat{S}_i(\succ_i): \mathcal{I}_i \to \bigcup_{I \in \mathcal{I}_i} A(I)$  as the mapping from information sets to actions defined by  $\hat{S}_i(\succ_i)(I) = S_{i,I}(\succ_i)(I)$  for each  $I \in \mathcal{I}_i$ ; that is,  $\hat{S}_i(\succ_i)$  is a standard game-theoretic strategy (complete contingent plan of action) defined by agent i selecting the action that is called for by the strategic plan  $S_{i,I}$  at information set I itself. For any  $S_{\mathcal{N},\mathcal{I}}$  and type realization  $\succ_{\mathcal{N}}$ , we can determine the terminal history and associated outcome that is reached when the game is played according to the profile of strategic collections  $S_{\mathcal{N},\mathcal{I}}(\succ_{\mathcal{N}})$  by following the profile of induced strategies  $\hat{S}_{\mathcal{N}}(\succ_{\mathcal{N}})$ . For each player i and type  $\succ_i$ , the induced strategy  $\hat{S}_i(\succ_i)$  also allows us to define the set of on-path information sets for a strategic collection. These are the information sets  $I \in \mathcal{I}_i$  such that there exist strategies for the other players and Nature such that I is on the path of play of  $\hat{S}_i(\succ_i)$ .

Induced strategies allow us to define equivalence of mechanisms: two mechanisms  $(\Gamma, S_{\mathcal{N},\mathcal{I}})$  and  $(\Gamma', S'_{\mathcal{N},\mathcal{I}})$  are *equivalent* if, for every profile of types  $\succ_{\mathcal{N}}$ , the distribution over outcomes from the induced strategies  $\hat{S}_{\mathcal{N}}(\succ_{\mathcal{N}})$  in  $\Gamma$  is the same as that from the induced strategies  $\hat{S}'_{\mathcal{N}}(\succ_{\mathcal{N}})$  in  $\Gamma'$ . This equivalence definition is purely outcome-based, and allows that  $(\Gamma, S_{\mathcal{N},\mathcal{I}})$  and  $(\Gamma', S'_{\mathcal{N},\mathcal{I}})$  have different planning horizons for the agents. Every mechanism implements a mapping from preference profiles to outcomes, which we call the *social choice rule*. If two mechanisms are equivalent, they implement the same social choice rule.

## 3.2. Simple Dominance

Strategic plan  $S_{i,I^*}(\succ_i)$  is *simply dominant at information set*  $I^*$  for type  $\succ_i$  of player i if the worst possible outcome for i in the continuation game assuming i follows  $S_{i,I^*}(\succ_i)(I)$  at all  $I \in \mathcal{I}_{i,I^*}$  is weakly preferred by i to the best possible outcome for i in the continuation game if i plays some other action  $a' \neq S_{i,I^*}(\succ_i)(I^*)$  at  $I^*$ . (We provide an example below to illustrate this definition.) We say that a strategic collection  $(S_{i,I^*}(\succ_i))_{I^*\in\mathcal{I}_i,\succ_i\in\mathcal{P}_i}$  is *simply dominant* if, for each type  $\succ_i \in \mathcal{P}_i$ , the strategic plan  $S_{i,I^*}(\succ_i)$  is simply dominant at  $I^*$  for each on-path information set  $I^*$ . We say that a game is simply dominant if it admits simply dominant strategies.

 $<sup>^{16}</sup>$ The assumption that  $\mathcal{I}_{i,I^*}\subseteq\mathcal{I}_i$  is what makes our simplicity standards dominance standards. Dropping this assumption and endowing players with beliefs of what other players do at simple information sets leads to an analogue of our theory for equilibria. A natural requirement on the collection of simple node sets is that if an agent classifies an information set  $I>I_1$  as simple from the perspective of information set  $I_1$ , then the agent continues to classify I as simple from the perspective of all information sets  $I_2>I_1$  such that  $I\geq I_2$ ; while we do not impose this requirement, it is satisfied in all of the examples of simple dominance that we study.

<sup>&</sup>lt;sup>17</sup>We focus on pure strategies; the extension to mixed strategies is straightforward.

<sup>&</sup>lt;sup>18</sup>When assessing  $S_{i,I^*}(\succ_i)(I)$ , we take the worst case over all game paths consistent with i following  $S_{i,I^*}(\succ_i)(I)$  at all  $I \in \mathcal{I}_{i,I^*}$ , and compare to the best case over *all* game paths following any alternative action  $a' \neq S_{i,I^*}(\succ_i)(I^*)$ . While formulated slightly differently than Li (2017b), who invoked the notion of an

Note that the collection of planning horizons,  $(\mathcal{I}_{i,I^*})_{I^*\in\mathcal{I}_i}$ , is a parameter of the model. In the sequel, we focus on planning horizons that vary agents' length of foresight. This is not necessary, however, and there are other ways to conceptualize what information sets are in the planning horizons. Given a fixed  $k \in \{0, 1, 2, ..., \infty\}$ , we say that agent i has k-step foresight if

$$\mathcal{I}_{i,I^*} = \{ I \in \mathcal{I}_i | I^* \le I \text{ and } I^* < I_1 < \dots < I_k < I \Rightarrow \exists \ell \in \{1,\dots,k\} \text{ s.t. } I_\ell \notin \mathcal{I}_i \}.$$

We refer to the resulting simply dominant strategic collections as k-step dominant and say that a strategy is k-simple if it is the induced strategy for some k-step dominant strategic collection. Varying k allows us to embed in our model the following special cases:

- $k = \infty$ : That is,  $\mathcal{I}_{i,I^*} = \{I \in \mathcal{I}_i | I^* \leq I\}$ , and i can plan all of her future moves. In this case, the induced strategy of any resulting strategic collection is obviously dominant in the sense of Li (2017b), and further, any obviously dominant strategy  $S_i$  in the sense of Li (2017b) determines an obviously dominant strategic collection  $(S_{i,I^*})_{I^* \in \mathcal{I}_i}$  in which  $S_{i,I^*}(I) = S_i(I)$  for any  $I^* \leq I$ . For these reasons, we refer to such strategic collections as *obviously dominant*. If a mechanism admits obviously dominant strategic collections, then we say it is *obviously strategy-proof* (OSP).
- k = 1: That is,  $\mathcal{I}_{i,I^*} = \{I \in \mathcal{I}_i | I^* \leq I \text{ and } I^* < I' < I \Rightarrow I' \notin \mathcal{I}_i\}$ , and i can plan one move ahead but not more. We refer to the resulting simply dominant strategic collections as *one-step dominant*. The information sets in  $\mathcal{I}_{i,I^*} \{I^*\}$  are called i's *next* information sets (from the perspective of  $I^*$ ). If a mechanism admits one-step dominant strategic collections, then we say it is *one-step simple (OSS)*.
- k = 0: That is  $\mathcal{I}_{i,I^*} = \{I^*\}$ , and i cannot plan any future moves. We refer to the resulting simply dominant strategic collections as *strongly obviously dominant*. In this case, we can also talk about strongly obviously dominant *strategies* because, as for obvious dominance, there is a one-to-one correspondence between strategic collections  $(S_{i,I^*})_{I^* \in \mathcal{I}_i}$  and the induced strategies  $\hat{S}_i(I^*) = S_{i,I^*}(I^*)$ . If a mechanism admits strongly obviously dominant strategic collections, then we say it is *strongly obviously strategy-proof (SOSP)*.

EXAMPLE—Simple Dominance in Ascending Auctions: We illustrate simple dominance by looking at an ascending auction for a single good. The payoff of an agent i is equal to the agent's value  $v_i$  minus their payment if the agent receives the good and it is equal to minus their payment otherwise. For the purposes of this example, an ascending auction is a finite game with the following properties. At each non-terminal information set, an agent is called to play and can take one of two possible actions: Stay In or Drop Out. Only agents who have not dropped out yet—called active agents—are called to play. Each non-terminal information set I is associated with the current price  $p(I) \ge 0$ , which weakly increases along each path of play. Whenever there is only one active agent left, the game ends, though it can also end when there are several active agents. One of the agents

earliest point of departure between two strategies, our definition is formally equivalent to his when  $\mathcal{I}_{i,I^*}$  is the set of all continuation information sets at which i moves. Both we and Li (2017b) require dominance only onpath; this choice is in line with, for example, Pearce's (1984) extensive-form rationalizability and Shimoji and Watson's (1998) conditional dominance. An alternative approach is to require simple dominance at all nodes (information sets) in the game, including off-path ones.

<sup>&</sup>lt;sup>19</sup>For instance, the planning horizon could consist of the information sets at which a measure of computational complexity of a decision problem is below some threshold; cf., for example, Arora and Barak (2009) for a survey of computational complexity criteria.

active when the game ends is designated the winner. The winner receives the good and pays the price associated with the last history at which this agent moved; a winner who has not yet moved pays 0. All other agents receive no good and pay 0.

The ascending auction is OSP because the strategy of staying in as long as the current price is below the agent's value is obviously dominant, as shown by Li (2017b). The ascending auction is also OSS because the following strategic collection is one-step dominant. For any information set  $I^*$  at which i moves and  $p(I^*) \le v_i$ , i's strategic plan is  $S_{i,I^*}(I^*) = \text{In and } S_{i,I^*}(I) = \text{Out for all next information sets } I > I^*; \text{ for any information}$ set  $I^*$  at which i moves and  $p(I^*) > v_i$ , the strategic plan is  $S_{i,I^*}(I) = \text{Out for } I = I^*$  and for all next information sets  $I > I^*$ . This strategic collection is one-step dominant because if  $p(I^*) \le v_i$ , then staying in at  $I^*$  and planning to drop out at the next information set gives a worst-case payoff of 0, which is no worse than dropping out now; if  $p(I^*) > v_i$ , then dropping out at  $I^*$  is weakly better than any scenario following staying in. The ascending auction might be SOSP, for instance when the starting prices are higher than the agents' values. In general, however, the ascending auction is not SOSP: Let i be the first mover and suppose the prices i might see along the path of the game start strictly below  $v_i$ , but prices further along the path are strictly above  $v_i$ . Then, at the first move of i, no move is strongly obviously dominant: the worst case from choosing In results in a strictly negative payoff, which is worse than choosing Out and getting 0. Thus, In is not strongly obviously dominant. Similarly, the payoff from choosing Out is 0, which is worse than choosing In and winning at the current low price, and so Out is not strongly obviously dominant, either.

REMARK 1—Plan updating and consistency: In the one-step dominant strategic collections in the example above, the action an agent plans at  $I^*$  for the next information set  $I > I^*$  may differ from the action the agent chooses upon actually reaching I, that is, we may have  $S_{i,I^*}(I) \neq S_{i,I}(I)$ . There is no need for such action updating in obviously dominant or strongly obviously dominant strategic collections. For each OSP collection, there is an equivalent OSP collection that is *consistent* in the following sense:  $S_{i,I^*}(I) = S_{i,I}(I)$  for all  $I \in \mathcal{I}_{i,I^*}$  and all  $I^* \in \mathcal{I}_i$ . SOSP collections are always consistent.

We emphasize that agents with inconsistent strategic plans are not necessarily time-inconsistent or irrational. Indeed, such agents might understand that they may adjust their plans later, and think of the partial strategic plan  $S_{i,I^*}$  as an argument establishing that playing  $S_{i,I^*}(I^*)$  is better than any other action they could take at  $I^*$ . The tentativeness of such partial plans is an important possibility in the under-explored game-theoretic paradigm of making choices as they arise, a paradigm that Savage (1954) described as "you can cross that bridge when you come to it" (cf. Introduction).

## 3.3. Simplicity Gradations and Design Flexibility

A direct verification shows that the smaller the planning horizon, the stronger is the resulting simplicity requirement. To formulate this result, for any planning horizons  $\mathcal{I}_{i,I^*}$  and  $\mathcal{I}'_{i,I^*}$  such that  $\mathcal{I}_{i,I^*} \subseteq \mathcal{I}'_{i,I^*}$ , we say that a strategic collection  $(S'_{i,I^*}(\succ_i))_{\succ_i \in \mathcal{P}_i}$  on  $\mathcal{I}'_{i,I^*}$  is an  $\mathcal{I}'_{i,I^*}$ -extension of a strategic collection  $(S_{i,I^*}(\succ_i))_{\succ_i \in \mathcal{P}_i}$  on  $\mathcal{I}_{i,I^*}$  if  $S'_{i,I^*}(\succ_i)(I) = S_{i,I^*}(\succ_i)(I)$  for all  $I \in \mathcal{I}_{i,I^*}$ .

THEOREM 1—Nesting of Simplicity Concepts: *If planning horizons*  $\mathcal{I}_{i,I^*}$  *and*  $\mathcal{I}'_{i,I^*}$  *are such that*  $\mathcal{I}_{i,I^*} \subseteq \mathcal{I}'_{i,I^*}$  *and strategic collection*  $S_{i,I^*}$  *is simply dominant at*  $I^*$  *for*  $\mathcal{I}_{i,I^*}$ , *then any*  $\mathcal{I}'_{i,I^*}$ -extension of  $S_{i,I^*}$  is simply dominant at  $I^*$  for  $\mathcal{I}'_{i,I^*}$ .

As a corollary, we conclude that the lower the parameter k, the more restrictive k-step simplicity becomes. Further, our class of simple dominance concepts has a natural lattice structure, with obvious dominance as its least demanding concept and strong obvious dominance as the most demanding one.

COROLLARY 1: (i) Take  $k, k' \in \{0, 1, 2, ..., \infty\}$  and assume k < k'. Then, any strategic collection that is k-step dominant is also k'-step dominant.

- (ii) If a strategic collection  $(S_{i,I^*})_{I^*\in\mathcal{I}_i}$  is simply dominant for some collection of simple information sets, then the induced strategy  $\hat{S}_i(I^*) = S_{i,I^*}(I^*)$  is obviously dominant.
- (iii) If the induced strategy  $\hat{S}_i(I^*) = S_{i,I^*}(I^*)$  is strongly obviously dominant, then the strategic collection is simply dominant for any  $(\mathcal{I}_{i,I^*})_{I^* \in \mathcal{I}_i}$ .

From an implementation perspective, an immediate consequence of Corollary 1 is that the set of k-step simple implementable social choice rules weakly expands as k is increased. The following result shows that in general, this inclusion is strict: that is, stronger simplicity constraints (lower k) reduce the flexibility of the designer.<sup>20</sup>

THEOREM 2—Simplicity-Flexibility Trade-off: Let  $k, k' \in \{0, 1, 2, ..., \infty\}$  and assume k' > k. There exist preference environments and social choice rules implementable in k'-step simple strategic collections, but not implementable in k-step simple strategic collections.

The presence of the simplicity-flexibility trade-off depends on the preference environment. For instance, Theorem 6 shows that in some environments, there is no loss in imposing one-step simplicity (k=1) relative to obvious strategy-proofness  $(k=\infty)$ : in these environments, any social choice rule that is OSP-implementable is also OSS-implementable.

To get a sense of why the inclusion can be strict, consider an environment with transfers in which there are at least two agents and each agent's value for an object comes from the same support with at least three distinct values. Suppose we want to allocate the object to the highest-value agent. This social choice rule can be implemented via an ascending auction and ascending auctions are OSS (we establish the one-step simplicity of ascending auctions in Theorem 6). At the same time, this social choice rule, and the price discovery it entails, cannot be implemented via SOSP mechanisms, which resemble posted prices (the posted price characterization of SOSP is given by our Theorem 8). For k, k' strictly larger than 0, the comparison is more subtle. Our proof in the Appendix constructs social rules that are k'-step simple implementable but not k-step simple implementable in no-transfer single-unit demand allocation environments.

### 3.4. Behavioral Microfoundations

We may think of simple strategic plans as providing guidance to a player that is unaffected even when they may be confused about the game they are playing, in the sense that they may mistake the game for a different game that has different players, actions, and precedence relations at non-simple information sets. An alternative interpretation is that the player is only given a partial description of the game: each time they are called to move, they are told what happens at their own simple information sets, but not at

<sup>&</sup>lt;sup>20</sup>In particular, the theorem shows that for any  $k < \infty$ , there are social choice rules that are OSP-implementable but not k-step implementable.

any other non-simple information set. If players have simply dominant strategic plans, the prediction of play is unaffected by the player's confusion or partial description of the game.

To formalize this idea, say that game  $\Gamma'$  is *indistinguishable from*  $\Gamma$  *from the perspective* of agent i at information set  $I^*$  of game  $\Gamma$  if there is an injection  $\lambda$  from the set of agent i's simple information sets  $\mathcal{I}_{i,I^*}$  in  $\Gamma$  into the set of agent i's information sets  $\mathcal{I}'_i$  in  $\Gamma'$  such that:

- 1. If  $I_1, I_2 \in \mathcal{I}_{i,I^*}$  and  $I_1$  precedes  $I_2$  in  $\Gamma$ , then  $\lambda(I_1)$  precedes  $\lambda(I_2)$  in  $\Gamma'$ .
- 2. For each  $I \in \mathcal{I}_{i,I^*}$ , there is a bijection  $\eta_I$  that maps actions at agent i's information set I in  $\Gamma$  onto actions at agent i's information set  $\lambda(I)$  in  $\Gamma'$ .
- 3. An outcome is possible following action a at  $I \in \mathcal{I}_{i,I^*}$  in  $\Gamma$  if and only if this outcome is possible following  $\eta_I(a)$  at  $\lambda(I)$  in  $\Gamma'$ .

We say that  $\lambda(I)$  is the game  $\Gamma'$  counterpart of information set I and  $\eta_I(a)$  is the game  $\Gamma'$  counterpart of action a at information set I in game  $\Gamma$ . The concept of indistinguishability captures the idea that agent i understands the precedence relation among simple information sets, as well as the available actions and possible outcomes at these information sets.

Simple dominance is equivalent to standard weak dominance on all games that are indistinguishable from the game played. We say that a strategy  $S_i$  of player i weakly dominates strategy  $S_i'$  in the continuation game beginning at  $I^*$  if following strategy  $S_i$  leads to weakly better outcomes for i than following strategy  $S_i'$ , irrespective of the strategies followed by other players. Note that here,  $S_i$  and  $S_i'$  denote full strategies in the standard game-theoretic sense of a complete contingent plan of action.

THEOREM 3—Behavioral Microfoundation: For each game  $\Gamma$ , agent i, type  $\succ_i$ , and collection of simple information sets  $(I_{i,I^*})_{I^* \in \mathcal{I}_i}$ , the strategic plan  $S_{i,I^*}$  is simply dominant from the perspective of  $I^* \in \mathcal{I}_i$  in  $\Gamma$  if and only if, in every game  $\Gamma'$  that is indistinguishable from  $\Gamma$  from the perspective of i at information set  $I^*$ , in the continuation game of  $\Gamma'$  starting at the counterpart of  $I^*$ , any strategy that at the counterpart of each  $I \in \mathcal{I}_{i,I^*}$  selects the counterpart of  $S_{i,I^*}(I)$  weakly dominates any strategy that does not select the counterpart of  $I^*$ .

When the strategic collection is consistent, this result says that the induced global strategy  $S_i(I) = S_{i,I}(I)$  is simply dominant in one game if and only if  $S_i(I)$  is weakly dominant in all indistinguishable games. When expressed in this way, this result corresponds to Li's (2017b) microfoundation for obvious strategy-proofness.<sup>21</sup>

# 3.5. Design Sufficiency of Perfect-Information Games

Under perfect information, each information set I contains a single history h and, to keep the notation at the minimum, we identify history h and information set  $\{h\}$ . The planning horizon at  $h^*$  then becomes the set  $\mathcal{H}_{i,h^*}$  of *simple histories* and the collection of planning horizons becomes  $(\mathcal{H}_{i,h^*})_{h^*\in\mathcal{H}_i}$ . We denote the corresponding strategic collections by  $(S_{i,h^*})_{h^*\in\mathcal{H}_i}$ .

Perfect-information games play a special role in designing simply dominant mechanisms because for any imperfect-information simply dominant mechanism, we can find

<sup>&</sup>lt;sup>21</sup>While the two results capture the same phenomenon, there is a slight difference between them even when restricted to OSP, as Li's (2017b) microfoundation assumes that  $\lambda$  is a bijection while we only require that  $\lambda$  is an injection. This difference has no impact on the validity of the claim nor the proofs.

an equivalent perfect-information one.<sup>22</sup> To make this point precise, for any imperfect-information game  $\Gamma$ , define the corresponding perfect-information game  $\Gamma'$  with the same set of histories as  $\Gamma$ . Given a collection of simple information sets  $(\mathcal{I}_{i,I^*})_{I^* \in \mathcal{I}_i}$  in  $\Gamma$ , we define the induced collection of simple histories  $(\mathcal{H}_{i,h^*})_{h^* \in \mathcal{H}_i}$  in  $\Gamma'$  such that  $\mathcal{H}_{i,h^*}$  consists of all histories in  $\mathcal{I}_{i,I^*}$ . For a strategic collection  $(S_{i,I^*})_{I^* \in \mathcal{I}_i}$ , we define the induced strategic collection  $(S_{i,h^*})_{h^* \in \mathcal{H}_i}$  such that  $S_{i,h^*}(h) = S_{i,I^*}(I)$ , where I is a continuation information set of  $I^*$ ,  $h^* \in I^*$  and  $h \in I$ .

THEOREM 4—Perfect-Information Reduction: If  $(S_{i,I^*})_{I^* \in \mathcal{I}_i}$  is simply dominant in an imperfect-information game  $\Gamma$  with simple information sets  $(\mathcal{I}_{i,I^*})_{I^* \in \mathcal{I}_i}$ , then in the corresponding perfect-information game  $\Gamma'$  with the induced simple histories  $(\mathcal{H}_{i,h^*})_{h^* \in \mathcal{H}_i}$ , the induced strategic collection  $(S_{i,h^*})_{h^* \in \mathcal{H}_i}$  is simply dominant.

To prove the theorem, consider an agent i with type  $\succ_i$ . Notice that if some history h is on-path for the strategic collection  $(\hat{S}_{i,h^*}(\succ_i))_{h^*\in\mathcal{H}_i}$  in  $\Gamma'$ , then the corresponding information set  $I\ni h$  is on-path for the strategic collection  $(\hat{S}_{i,l^*}(\succ_i))_{I^*\in\mathcal{I}_i}$  in  $\Gamma$ . Furthermore, the worst outcome following  $S_{i,h^*}(h)=S_{i,l^*}(I)$  in  $\Gamma'$  is weakly better than the worst outcome over the entire information set I when following this strategy. Similarly, the best outcome following an alternative action  $a\neq S_{i,h^*}(h)$  at h is worse than the best outcome following an alternative action  $a\neq S_{i,h^*}(h)$  over the entire information set h. Thus, if the strategic plan  $S_{i,l^*}(I)$  is simply dominant in  $\Gamma$ , then the induced strategic plan  $S_{i,h^*}(\succ_i)$  is simply dominant in  $\Gamma'$ .

In light of Theorem 4, the restriction to perfect-information games does not affect the class of social choice rules that can be implemented in simple strategies. We hence adapt this restriction in the study of mechanism design in the next two sections.

#### 4. CHARACTERIZING SIMPLE MECHANISMS

We now consider three special cases of the above simplicity standards—obvious dominance, one-step dominance, and strong obvious dominance—and characterize simple mechanisms and social rules in environments both with and without transfers. To make our analysis relevant for market-design applications and to avoid general impossibility results such as the Gibbard–Satterthwaite theorem, we must allow some restrictions on the domains of agent preferences. We formalize this as follows.

We take as a primitive a *structural dominance relation* over outcomes, denoted  $\trianglerighteq$ , where  $\trianglerighteq$  is a reflexive and transitive binary relation on  $\mathcal{X}$ . The notation  $x \trianglerighteq y$  is read as "x weakly dominates y." If  $x \trianglerighteq y$  but not  $y \trianglerighteq x$ , then we write  $x \trianglerighteq y$ , and say that "x strictly dominates y." For instance, in environments with transfers, outcome x dominates outcome y for an agent if the agent receives a higher transfer under outcome x, and all else is equal. We say that a preference ranking  $\succsim_i$  is *consistent* with  $\trianglerighteq$  if  $x \trianglerighteq y$  implies that  $x \succsim_i y$  and  $x \trianglerighteq y$  implies that  $x \succ_i y$ .

<sup>&</sup>lt;sup>22</sup>An analogous property of obvious strategy-proofness was first asserted in a footnote in Ashlagi and Gonczarowski (2018). Following our work, Mackenzie (2020) extended this property of obvious strategy-proofness to extensive-form games without perfect recall.

<sup>&</sup>lt;sup>23</sup>For brevity, we write "weakly dominates" rather than "weakly structurally dominates" when the context makes clear that we refer to outcomes, comparable in terms of  $\triangleright$ , and not to strategies, comparable in the game-theoretic sense of weak dominance.

We allow the possibility that different agents have different dominance relations,  $\trianglerighteq_i$ , and therefore different preference domains. We assume that all rankings in  $\mathcal{P}_i$  are consistent with  $\trianglerighteq_i$ . If  $x \trianglerighteq_i y$  and  $y \trianglerighteq_i x$ , then x and y are  $\trianglerighteq_i$ -equivalent. Any  $\trianglerighteq_i$  determines an equivalence partition of  $\mathcal{X}$ . We refer to each element  $[x]_i = \{y \in \mathcal{X} : x \trianglerighteq_i y \text{ and } y \trianglerighteq_i x\}$  of the equivalence partition as a payoff. Consistency implies that each preference ranking in  $\mathcal{P}_i$  induces a well-defined preference ranking over payoffs in the natural way:  $[x]_i \succsim_i [y]_i$  if  $x \succsim_i y$  and  $[x]_i \succ_i [y]_i$  if  $x \succ_i y$ . To avoid unnecessary formalism, we use the same symbol for preferences over payoffs as for preferences over outcomes, and write "payoff x" for  $[x]_i$  and phrases such as "payoff x obtains" when the realized outcome belongs to  $[x]_i$ . Unless stated otherwise, we assume in this section that the preference domain  $\mathcal{P}_i$  is rich in the following sense: the set of induced preferences over payoffs consists of all strict rankings over payoffs.

The framework of rich preference domains is flexible and encompasses many standard economic environments. Some examples of rich domains will help clarify the definitions and notation:

- *Voting*: Every agent has strict preferences over all alternatives in  $\mathcal{X}$ . This is captured by the trivial dominance relation  $\trianglerighteq_i$  in which  $x \trianglerighteq_i y$  implies x = y for all i. Each agent's preference domain  $\mathcal{P}_i$  partitions  $\mathcal{X}$  into  $|\mathcal{X}|$  individual payoffs. Richness implies that each  $\mathcal{P}_i$  consists of all strict preference rankings over  $\mathcal{X}$ .
- Allocating indivisible goods without transfers: Each  $x \in \mathcal{X}$  describes the entire allocation of goods to each of the agents. Each agent has strict preferences over each bundle of goods she may receive, but is indifferent over how goods she does not receive are assigned to others. This is captured by a dominance relation  $\trianglerighteq_i$  for agent i defined as follows:  $x \trianglerighteq_i y$  if and only if agent i receives the same set of goods in outcomes x and y. Each payoff of agent i can be identified with the set of objects she receives. Richness implies that every strict ranking of these sets belongs to  $\mathcal{P}_i$  for each i.

With these two examples in mind, we say that an environment is *without transfers* if the dominance relation  $\trianglerighteq_i$  is symmetric for all i. Non-symmetric dominance relations  $\trianglerighteq_i$  allow us to model transfers: all else equal, having more money dominates having less. Examples of rich domains with transfers include the following:

- Social choice with transfers: Let X = Y × W<sup>N</sup>, where Y is a set of substantive outcomes and W ⊆ R a (finite) set of possible transfers. For a fixed y ∈ Y, agent i prefers to pay less rather than more and is indifferent between any two outcomes that vary only in other agents' transfers. The structural dominance relation is (y, w) ⊵<sub>i</sub> (y', w') if and only if y = y' and w<sub>i</sub> ≥ w'<sub>i</sub>, where w ≡ (w<sub>i</sub>)<sub>i∈N</sub> is the profile of transfers.
  Auctions: Let X ⊆ N<sup>O</sup> × W<sup>N</sup>, where O is a finite set of goods and W ⊆ R is a finite
- Auctions: Let  $\mathcal{X} \subseteq \mathcal{N}^{\mathcal{O}} \times \mathcal{W}^{\mathcal{N}}$ , where  $\mathcal{O}$  is a finite set of goods and  $\mathcal{W} \subsetneq \mathbb{R}$  is a finite set of transfers. Each agent *i* prefers to win more goods and to pay less rather than

<sup>&</sup>lt;sup>24</sup>Our use of the term richness shares with other uses of the term in the literature the idea that the domain of preferences contains sufficiently many profiles: if certain preference profiles belong to the domain, then some other profiles belong to it as well (cf. Dasgupta, Hammond, and Maskin (1979) and Pycia (2012)). The more outcome pairs that are comparable by the structural dominance relation  $\triangleright_i$ , the smaller the resulting preference domain and less restrictive the simple dominance requirement. At one extreme,  $\triangleright_i$  is an identity relation for each  $i \in \mathcal{N}$ , agents' preference domains consist of all strict rankings, and simple mechanisms resemble dictatorships as in Gibbard (1973) and Satterthwaite (1975) and our Corollary 2. At the other extreme,  $\triangleright_i$  compares all outcomes, each agent is indifferent among all outcomes, and any strategy in any game is simple. In between these extremes, we have other classes of simple mechanisms, as we explore in this section. We would like to thank referees for these clarifications.

<sup>&</sup>lt;sup>25</sup>A binary relation  $\geq_i$  is *symmetric* if  $x \geq_i y$  implies  $y \geq_i x$ . It is easy to see that this holds in the examples without transfers above, but not in those with transfers below.

more. Denoting by  $O_i$  the set of goods allocated to i and writing  $O = (O_i)_{i \in \mathcal{N}}$ , the structural dominance relation is given by  $(O; w) \trianglerighteq_i (O'; w')$  if and only if  $O_i \supseteq O'_i$  and  $w_i \ge w'_i$ .

These are just a few examples of settings that fit into our general model. While richness is a flexible assumption, not all preference domains are rich. For instance, domains of single-peaked preferences are typically not rich. Arribillaga, Massó, and Neme (2020) showed that our millipede construction does not extend to single-peaked preference domains.

#### 4.1. Obvious Dominance

By Theorem 1, the weakest simplicity standard in our class is obvious dominance of Li (2017b). Recall that, in analyzing obvious dominance, we do not need to distinguish between strategies and strategic plans; thus, for simplicity of exposition, we focus on strategies in this section. If a game  $\Gamma$  admits a profile of obviously dominant strategies, then the game and the resulting mechanism are said to be *obviously strategy-proof (OSP)*.

In this section, we focus on environments without transfers and show that any OSP game is equivalent to a *millipede game*.<sup>26</sup> Roughly speaking, a millipede game is a clinchor-pass game similar to a centipede game (Rosenthal (1981)), but with possibly more players and more actions ("legs") at each node. A simple example of a millipede game in an object allocation environment is a *serial dictatorship* in which there are no passing moves and all payoffs that are not precluded by the earlier choices of other agents are clinchable (cf. Section 4.3).

As a preliminary step to define millipede games, we introduce the following definitions, which apply to any game  $\Gamma$ . Given some history h, we say that payoff x is *possible* for agent i at h if there is a terminal history  $\bar{h} \supseteq h$  at which agent i obtains payoff x. We use  $P_i(h)$  to denote the set of possible payoffs for i at h. We say that agent i has *clinched* payoff x at history h if, at all terminal histories  $\bar{h} \supseteq h$ , agent i receives payoff x. If i moves at h, takes action  $a \in A(h)$ , and has clinched x at the history (h, a), then we call action a a *clinching action*; any action at h that is not a clinching action is called a *passing action*. We denote by  $C_i(h)$  the set of all payoffs x that are *clinchable* for i at h; that is,  $C_i(h)$  is the set of payoffs for which there is an action  $a \in A(h)$  such that i has clinched x at the history (h, a). At a terminal history  $\bar{h}$ , no agent is called to move and there are no actions; however, it is notationally convenient to define  $C_i(\bar{h}) = \{x\}$ , where x is the payoff that i obtains at terminal history  $\bar{h}$ .

We further define  $C_i^{\subseteq}(h) = \{x : x \in C_i(h') \text{ for some } h' \subseteq h \text{ s.t. } i_{h'} = i\}$  to be the set of payoffs that i can clinch at some subhistory of h, and  $C_i^{\subseteq}(h) = \{x : x \in C_i(h') \text{ for some } h' \subseteq h \text{ s.t. } i_{h'} = i\}$  to be the set of payoffs that i can clinch at some strict subhistory of h. Note that while the definition of  $C_i(h)$  presumes that i moves at h or h is terminal, the payoff sets  $P_i(h)$ ,  $C_i^{\subseteq}(h)$  and  $C_i^{\subseteq}(h)$  are well-defined for any h, whether i moves at i or not, and whether i is terminal or not. Finally, consider a history i such that  $i_{h'} = i$  for some i and either i is a terminal history. We say that payoff i becomes impossible for i at i if i i

Given a mechanism  $(\Gamma, S_N)$  and a type  $\succ_i$ , we call strategy  $S_i(\succ_i)$  a greedy strategy if, at any history  $h \in \mathcal{H}_i$ , it satisfies the following: if the  $\succ_i$ -best still-possible payoff in  $P_i(h)$ 

<sup>&</sup>lt;sup>26</sup>This characterization complements Li's (2017b) result that in binary allocation environments with transfers, every OSP mechanism is equivalent to a personal clock auction; cf. Section 4.2.

is clinchable at h, then  $S_i(\succ_i)(h)$  clinches this payoff; otherwise,  $S_i(\succ_i)(h)$  is a passing action. A greedy strategic plan is defined in the same way.<sup>27</sup>

Given these definitions, we define a *millipede game* as a finite extensive-form game of perfect information that satisfies the following properties:

- 1. Nature either moves once, at the empty history  $h_{\emptyset}$ , or Nature has no moves.
- 2. At any history at which an agent moves, all but at most one action are clinching actions, and following any clinching action, the agent does not move again.
- 3. At all h, if there exists a previously unclinchable payoff x that becomes impossible for agent  $i_h$  at h, then  $C_{i_h}^{\subset}(h) \subseteq C_{i_h}(h)$ .

We refer to millipede games with greedy strategies as *millipede mechanisms*. In a millipede game, it is obviously dominant for an agent to clinch the best possible payoff at h whenever it is clinchable. The last condition of the millipede definition ensures that passing at h is obviously dominant when an agent's best possible payoff at h is not clinchable.

THEOREM 5—Millipedes: Consider an environment without transfers. Every OSP mechanism is equivalent to a millipede mechanism. Every millipede mechanism is OSP.

This theorem is applicable in many environments. This includes allocation problems in which agents care only about the object(s) they receive, in which case clinching actions correspond to taking a specified (set of) object(s) and leaving the remaining objects to be distributed amongst the remaining agents. Theorem 5 also applies to standard social choice problems in which no agent is indifferent between any two outcomes (e.g., voting), in which case clinching corresponds to determining the final outcome for all agents. In such environments, we have the following:

COROLLARY 2: Let each agent's preference domain  $P_i$  be the space of all strict rankings over outcomes X. Then, every OSP game is equivalent to a game in which either:

- (i) the first agent to move can clinch any possible outcome and has no passing action; or
- (ii) there are only two outcomes that are possible when the first agent moves, and the first mover can either clinch any of them, or can clinch one of them or pass to a second agent, who is presented with an analogous choice, etc.

The former case of Corollary 2 is the standard dictatorship, with a possibly restricted set of outcomes. The latter case is a generalization that allows an agent to enforce one of the two outcomes, but not the other, at her turn; see Figure 1 for an example. In particular, this corollary gives an analogue of the Gibbard–Satterthwaite dictatorship result, with no efficiency assumption.

The full proof of Theorem 5 is in the Appendix; here, we provide a brief sketch of the more interesting direction that, for any OSP game  $\Gamma$ , there is an equivalent millipede game. We construct this millipede game via the following transformations. Starting with any arbitrary game, we begin by breaking information sets; this only shrinks the set of possible outcomes any time an agent is called to play, which preserves the min/max obvious dominance inequality. For similar reasons, we can shift all of Nature's moves to the beginning of the game, and so now have a perfect-information game  $\Gamma'$  in which Nature

 $<sup>^{27}</sup>$ A stronger concept of a greedy strategy would additionally require that when passing, the agent takes an action a such that they are indifferent between the best possible payoffs at h and (h, a). (Such an action a exists because  $P_i(h) = \bigcup_{a \in A(h)} P_i((h, a))$ .) This distinction is immaterial for millipede games, since they have at most one passing action at each history, and all of our results are valid for both concepts of greediness.

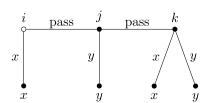


FIGURE 1.—An example of a non-dictatorial millipede game in a voting environment with two outcomes,  $\mathcal{X} = \{x, y\}$ . The obviously dominant (greedy) strategy profile is for any agent to clinch if she is offered to clinch her preferred option among  $\{x, y\}$ , and otherwise pass.

moves once, as the first mover. Second, if there are two passing actions a and a' at some on-path history h, then there are (by definition) at least two payoffs that are possible for i following each. We show that obvious dominance then implies that i must have some continuation strategy that can guarantee his top possible payoff in the continuation game following at least one of a or a'. Then, we can construct an equivalent game via a transformation in which we add an action that allows i to clinch this payoff already at h by making all such "future choices" today. We also rely on Li's pruning, in which the actions no type chooses are removed from the game tree; cf. Appendix A.1. We repeat these transformations until there is at most one passing action remaining. The final step of the proof is to show that these transformations give us a millipede game. This last step relies on richness and shows that if there remains some h such that agent i cannot clinch her favorite possible payoff at h, the game must promise i that she will never be strictly worse off by passing, which is condition 3.

### 4.2. One-Step Dominance

One-step simple dominance is stronger than obvious dominance. To see why this strengthening might be useful, recall that obviously dominant strategies may not be intuitively simple; an already discussed stark example is White's winning strategy in chess. As another example, consider a no-transfer object allocation environment and the two-player millipede game in Figure 2. At the first move, type  $o_{100} \succ_i o_1 \succ_i o_2 \succ_i \cdots \succ_i o_{99}$  is offered her second-favorite object,  $o_1$ , while her top choice,  $o_{100}$ , is possible. The obviously dominant greedy strategy of this type is to pass; however, if she does so, she may not be offered the opportunity to clinch her top object,  $o_{100}$ , or even go back to her second-best object,  $o_1$ , until far into the future. Thus, while passing is obviously dominant, comprehending this requires the ability to reason far into the future of the game and to perform lengthy backward induction.<sup>29</sup>

The more demanding concept of one-step simplicity eliminates the intuitively complex, yet still formally obviously dominant, strategies such as White's winning strategy in chess and the greedy strategy in the millipede of Figure 2, while still classifying greedy strategies in serial dictatorships and ascending auctions as simple.

<sup>&</sup>lt;sup>28</sup>Both parts of this transformation were first asserted for OSP in a footnote by Ashlagi and Gonczarowski (2018); cf. our Theorem 4 and Lemma A.4.

<sup>&</sup>lt;sup>29</sup>The first 100 moves of this millipede cannot be substantially shortened because, given the players' greedy strategies, for k = 1, ..., 50, i can obtain  $o_{k+1}$  if and only if j's top choice is  $o_{100-k+1}$  or a lower-indexed object, and j can obtain  $o_{100-k+1}$  if and only if i's top choice is  $o_k$  or a higher-indexed object.

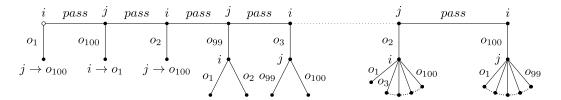


FIGURE 2.—An example of a millipede game with two agents  $\{i, j\}$  and 100 objects  $\{o_1, o_2, \dots, o_{100}\}$ . If the first clinching is in an agent's first 50 moves, then the other agent is given the choice of clinching any object he or she could have clinched previously; if the first clinching is after the clinching agent's first 50 moves, then the other agent is given the choice of clinching any still-available object.

## Binary Allocation With Transfers

Consider a set of outcomes  $\mathcal{X} = Y \times \mathbb{R}^{\mathcal{N}}$ , where  $Y \subseteq \{0, 1\}^{\mathcal{N}}$  is a set of feasible allocations and  $\mathbb{R}^{\mathcal{N}}$  is the set of profiles of transfers, one for each agent. A generic allocation is denoted y and a generic profile of transfers  $w = (w_i)_{i \in \mathcal{N}}$ . Agents have types  $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$ , where  $0 \le \underline{\theta}_i < \overline{\theta}_i < \infty$ , and each agent's preferences are represented by a quasilinear utility function:  $u_i(\theta_i, y, w) = \theta_i y_i + w_i$ . Following Li (2017b), we call this preference environment binary allocation with transfers.<sup>30</sup> This framework captures many important environments of economic interest, including single-unit auctions, procurement auctions, and binary public goods games.

For these environments, Li introduced the class of *personal clock auctions*, which generalize the ascending auction in several ways: agents may face different individualized prices ("clocks"); at any point, there may be multiple quitting actions that allow agents to drop out of the auction, or multiple continuing actions that allow them to stay in the auction; and when an agent quits, her transfer need not be zero. The key restrictions are that each agent's clock must be monotonic, and that whenever the personal price an agent faces strictly changes, she must be offered an opportunity to quit. The formal definition of a personal clock auction can be found in Appendix B.3 of the Supplemental Material (Pycia and Troyan (2023)), where we also prove Theorem 6.

Li (2017b) showed that in binary allocation settings, OSP games are equivalent to personal clock auctions. We strengthen this result to show that personal clock auctions are also OSS. Thus, in the binary setting, there is no loss in imposing one-step dominance: any OSP-implementable social choice rule is also implementable in one-step dominant strategic collections.

THEOREM 6—OSS and Personal Clock Auctions: *In binary allocation settings with transfers*, *every one-step simple mechanism is equivalent to a personal clock auction with one-step dominant strategic collections*. *Furthermore*, *every personal clock auction is one-step simple*.

Because our Corollary 1 shows that any OSS mechanism is also OSP, the first part of the theorem follows from Li's (2017b) result that any OSP mechanism is equivalent to a personal clock auction with greedy strategies, provided we can find a profile of one-step dominant strategic collections that replicates the play of Li's greedy strategies. We construct these collections in the proof of the second part of the theorem by generalizing the

<sup>&</sup>lt;sup>30</sup>We allow for a continuum of types and transfers here in order to reproduce the binary allocation environment of Li (2017b). Our simplicity concepts extend to this environment when we substitute inf for min and sup for max in our definitions. Richness plays no role in the binary allocation results.

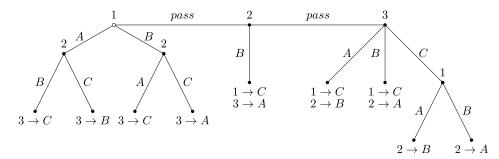


FIGURE 3.—An example of a monotonic millipede game. The game allocates three objects A, B, and C to three agents (or players) 1, 2, and 3. Agent 1 moves first and can clinch one of the objects A or B, or can pass. The second move is made by agent 2, who either clinches an object (in which case the allocation is fully determined) or passes (the passing move is only possible following a pass by 1). Agent 3 only moves following two passes; this player can then clinch any object. If agent 3 clinches A or B, then the allocation is determined, and if agent 3 clinches C, then agent 1 can choose between A and B.

one-step dominant strategic plans from the ascending auction example of Section 3. As in the ascending auction, in a personal clock auction, whenever an agent's price changes, she is offered an opportunity to drop out. In effect, the strategic plan to stay in whenever an agent's personal price is below her valuation and to plan to drop out at any next information set is one-step dominant.

## **Environments Without Transfers**

Recall the complex millipede game of Figure 2 that requires lengthy foresight on the part of the agents. One-step simplicity eliminates these complex millipede games, and leaves only millipedes that are monotonic in the following sense: a millipede game  $\Gamma$  is *monotonic* if, for any agent i and any histories h, h' such that:  $(h, a^*) \subseteq h'$  where  $a^*$  is a passing action at h,  $i_h = i$ ,  $i_{h'} = i$  or h' is terminal, and  $i_{h''} \neq i$  for any h'' such that  $h \subseteq h'' \subseteq h'$ , either (i)  $C_i(h) \subseteq C_i(h')$  or (ii)  $P_i(h) \setminus C_i(h) \subseteq C_i(h')$ . In words, this says that if an agent passes at h, the next time she moves, she is offered to clinch either (i) everything she could have clinched at h or (ii) everything that was possible, but not clinchable, at h. Some millipede games, such as serial dictatorships in which each agent only moves once and has no passing action, are trivially monotonic; for a less trivial example of a monotonic millipede game that allows for passing actions and more complex allocation rules, see Figure 3. We say that a mechanism is monotonic when the underlying game is.

THEOREM 7—Monotonic Millipedes: In environments without transfers, every one-step simple millipede mechanism is equivalent to a monotonic millipede mechanism with one-step dominant strategic collections. Furthermore, every monotonic millipede mechanism is one-step simple.

At any history h in a monotonic millipede game, the one-step dominant strategic plan is as follows: if the agent can clinch her top outcome that is possible at h, then she does so; otherwise, the agent passes at h, and for any next history h', the strategic plan is to clinch her top possible outcome in  $C_i(h')$ . If clause (i) of monotonicity holds, then this is at least as good as anything she could clinch at h (since the clinchable set weakly expands); if clause (ii) of monotonicity holds, then she obtains her best possible payoff in  $P_i(h)$ , which is again at least as good as anything that was clinchable at h.

From the perspective of an agent playing in a game, monotonic games seem particularly simple: each time an agent is called to move, she knows that if she chooses to pass, at her next move, she will either be able to clinch everything she is offered to clinch currently, or she will be able to clinch everything possible but currently unclinchable. On the other hand, in a non-monotonic game such as that in Figure 2, an agent's possible clinching options may be strictly worse for many moves in the future, before eventually the agent is re-offered what she was able to clinch in the past (or something better). If agents are unable to plan far ahead in the game tree, it may be difficult to recognize that passing is obviously dominant in such a game; in a monotonic game, however, agents only need to be able to plan at most one step at a time to recognize that passing is a dominant choice.

Further, from a practical implementation perspective, monotonic games are also particularly simple for a designer to run dynamically: at each step, the designer only need tell an agent her possible clinching options today, plus that if she passes, at her very next move, her clinchable set will either weakly expand, or she will be offered everything possible that she was not offered today. Such a partial, one-step-at-a-time description is simpler than trying to describe all of the possibilities many moves in the future that would be necessary to implement more complex, non-monotonic OSP games.

## 4.3. Strong Obvious Dominance, Choice Mechanisms, and Posted Prices

In light of Theorem 1, the strongest simplicity standard in our class is strong obvious dominance. If a game  $\Gamma$  admits a profile of strongly obviously dominant strategic collections, we say that it is *strongly obviously strategy-proof (SOSP)*. Random Priority is SOSP,<sup>31</sup> but ascending auctions are not. Thus, SOSP further delineates the class of games that are simple to play, by eliminating millipede games that require even one-step forward-looking behavior. As there is a one-to-one correspondence between strongly obviously dominant strategie collections and strongly obviously dominant strategies, for simplicity of exposition we focus on strategies. Additionally, in this section, we make use of the concept of an undominated payoff, where we say that a payoff x is *undominated* in a subset of payoffs for agent i if there is no payoff y in this subset such that  $y \triangleright_i x$ . A mechanism  $(\Gamma, (S_i(\succ_i))_{i \in \mathcal{N}})$  is *pruned* if every information set in  $\Gamma$  is on the path of play for some type of some player. Li (2017b) observed that every OSP mechanism is equivalent to a pruned OSP mechanism. The same is true for SOSP; cf. Appendix A.1.

Strongly obviously strategy-proof games are particularly simple to play. Any strongly obviously dominant strategy is greedy. Further, SOSP games can be implemented so that each agent is called to move at most once and has at most one history at which her choice of action is payoff-relevant. Formally, we say a history h at which agent i moves is payoff-irrelevant for this agent if i receives the same payoff at all terminal histories  $\bar{h} \supset h$ ; if i moves at h and this history is not payoff-irrelevant, then it is *payoff-relevant* for i. The definition of SOSP and richness of the preference domain give us the following.

LEMMA 1: Along each game path of a pruned SOSP mechanism, there is at most one payoff-relevant history for each agent.

<sup>&</sup>lt;sup>31</sup>By Random Priority we mean the following mechanism that is commonly used to allocate indivisible objects to a group of agents: first, Nature randomly selecting an ordering of the agents, and then the agents are called one at a time in this order to select their favorite still-remaining object at their turn (cf. Abdulkadiroğlu and Sönmez (1998), Bogomolnaia and Moulin (2001), and Liu and Pycia (2016) who studied the direct mechanism implementation of Random Priority). This mechanism is also sometimes called Random Serial Dictatorship, and is a special case of the more general sequential choice mechanisms we characterize in this section.

This result—proven in Appendix B.5 of the Supplemental Material—allows us to further conclude that, for a given game path, the unique payoff-relevant history (if it exists) is the first history at which an agent is called to move. While an agent might be called to act later in the game, and her choice might influence the continuation game and the payoffs for other agents, it cannot affect her own payoff.

Building on Lemma 1, we show that SOSP effectively implies that agents—in a sequence—are faced with choices from personalized menus; for example, in allocation with transfers, this may be menus of object-price pairs. At the typical payoff-relevant history, an agent is offered a menu of payoffs that she can clinch, she selects one of the alternatives from the menu, and she is never called to move again. More formally, we say that  $\Gamma$  is a sequential choice game if it is a perfect-information game in which Nature moves first, if at all. The agents then move sequentially, with each agent called to play at most once. The ordering of the agents and the sets of possible outcomes at each history are determined by Nature's action and the actions taken by earlier agents. As long as there are either at least three distinct undominated payoffs possible for the agent who is called to move or there is exactly one such payoff, the agent can clinch any of the possible payoffs. When exactly two undominated payoffs are possible for the agent who moves, the agent can be faced with either (i) a set of clinching actions that allow the agent to clinch either of the two payoffs, (ii) a passing action and a set of clinching actions that allow the agent to clinch exactly one of these payoffs. Note that we allow potentially many ways of clinching the same payoff; we can conceptualize the many ways of clinching a fixed payoff as clinching it and sending a message from a predetermined set of messages. Note also that (ii) does not allow the agent to clinch the other pavoff.

THEOREM 8—Sequential Choice: Every strongly obviously strategy-proof mechanism is equivalent to a sequential choice mechanism with greedy strategies. Every sequential choice mechanism with greedy strategies is strongly obviously strategy-proof.

Theorem 8 applies to any rich preference environment, including both those with and without transfers. In an object allocation model without transfers, every SOSP mechanism resembles a priority mechanism (or, sequential dictatorship), in which agents are called sequentially and offered to clinch any object that still can be clinched given earlier clinching choices; they pick their most preferred object and leave the game. The key difference between a sequential choice game and priority mechanisms is that at an agent's turn in sequential choice, she need not be offered all still-available objects.

In environments with transfers, sequential choice games can be interpreted as sequential posted-price games. In a binary allocation setting with a single good and transfers, each agent is approached one at a time, and given a take-it-or-leave-it (TIOLI) offer of a price at which she can purchase the good; if an agent refuses, the next agent is approached, and given a (possibly different) TIOLI offer, etc. If there are multiple objects for sale, each agent is offered a menu consisting of several bundles of objects with associated prices, and selects her most preferred option from the menu.

Price mechanisms are ubiquitous in practice. Even on eBay, which began as an auction website, Einay, Farronato, Levin, and Sundaresan (2018) documented a dramatic shift in the 2000s from auctions to posted prices as the predominant selling mechanism. Posted prices have also garnered significant attention in the computer science community. For instance, computing the optimal allocation in a combinatorial Vickrey auction can be complex even from a computational perspective, and several papers have shown good performance using sequential posted-price mechanisms (e.g., Chawla, Hartline, Malec, Sivan

(2010) and Feldman, Gravin, and Lucier (2014)). By formalizing a strategic simplicity-based explanation for the popularity of these mechanisms, our Theorem 8 complements this literature.<sup>32</sup>

### 5. CONCLUSION

We study the question of what makes a game simple to play, and introduce a general class of simplicity standards that vary the planning horizons of agents in extensive-form imperfect-information games. We allow agents that form a strategic plan only for a limited horizon in the continuation game, and the agents may update these plans as the game progresses and the future becomes the present. The least restrictive simplicity standard included in our class is Li's (2017b) obvious strategy-proofness, which presumes agents have unlimited foresight of their own actions, while the strongest, strong obvious strategy-proofness, presumes no foresight. For each of these standards, as well as an intermediate standard of one-step simplicity, we provide characterizations of simple mechanisms in various environments with and without transfers, and show that our simplicity standards delineate classes of mechanisms that are commonly observed in practice. We show that SOSP delineates a class of posted-price mechanisms, OSS delineates a class of ascending clock mechanisms, and OSP delineates a richer class of mechanisms we call millipedes.<sup>33</sup> Along the way, we provide a logically consistent—though limited to simple games—approach to the analysis of agents with limited foresight.

Our results contribute to the understanding of the fundamental trade-off between simplicity of mechanisms and the ability to implement other social objectives, such as efficiency and revenues. In environments with transfers, Vickrey (1961), Riley and Samuelson (1981), Myerson (1981), Manelli and Vincent (2010), and Gershkov, Goeree, Kushnir, Moldovanu, and Shi (2013) showed that the efficiency and revenues achieved with Bayesian implementation can be replicated in dominant strategies; thus, the accompanying increase in simplicity may come without efficiency and revenue costs. Li (2017b) and our paper advance this insight further and establish that obviously strategy-proof and one-step simple mechanisms can also implement efficient outcomes (and revenue-maximizing outcomes).<sup>34</sup> At the same time, strong obvious dominance is more restrictive, and more severely limits the class of implementable objectives. In environments with transfers, SOSP generally precludes efficiency and revenue maximization.<sup>35</sup> In environments without transfers, however, even SOSP mechanisms—serial dictatorships—can achieve efficient outcomes. Building on the results of the present paper, in single-unit demand allocation problems without transfers, Pycia and Troyan (2023) and Pycia (2019)

<sup>&</sup>lt;sup>32</sup>Prior economic studies on the focal role of posted prices in mechanism design—for example, Hagerty and Rogerson (1987) and Copic and Ponsati (2016)—focused on bilateral trade, while our analysis is applicable to any economic environment satisfying our richness assumption.

<sup>&</sup>lt;sup>33</sup>Building on these results, Pycia and Troyan (2023) showed that every obviously strategy-proof, Pareto efficient, and symmetric mechanism is equivalent to Random Priority.

<sup>&</sup>lt;sup>34</sup>In Li's binary allocation settings, we show that all OSP mechanisms can be simplified to OSS.

<sup>&</sup>lt;sup>35</sup>For instance, when we want to allocate an object to the highest value agent with transfers and with at least two agents, and agents' values are drawn i.i.d. from among at least three values, an impossibility result obtains: no SOSP and efficient mechanism exists. This is implied by Theorem 8. This also shows that SOSP mechanisms raise less revenue than optimal auctions. On the other hand, Armstrong (1996) showed that posted prices achieve good revenues when bundling allows the seller to equalize the valuations of buyers, and Chawla et al. (2010) and Feldman, Gravin, and Lucier (2014) showed that sequential price mechanisms achieve decent revenues even without the bundling/equalization assumption.

showed that the restriction to strongly obvious strategy-proof mechanisms allows the designer to achieve virtually the same efficiency and many other objectives as those achievable in merely strategy-proof mechanisms. Thus, in many environments, simplicity entails no efficiency loss. In other environments, the trade-off between simplicity and efficiency is more subtle. Our Theorem 2 shows that, in general, imposing more restrictive simplicity standards on the mechanisms limits the set of implementable social choice functions.<sup>36</sup>

Our work is complementary to the experimental literature on how mechanism participants behave and what elements of design enable them to play equilibrium strategies; cf., for example, Kagel, Harstad, and Levin (1987) and Li (2017b). While this literature identifies implementation features that facilitate play and confirms that obviously strategy-proof mechanisms are indeed simpler to play than merely strategy-proof mechanisms, while strongly obviously strategy-proof mechanisms are easier still and nearly all participants play them as expected (see Bo and Hakimov (2020) and Chakraborty and Kendall (2022)),<sup>37</sup> our general theory of simplicity opens new avenues for experimental investigations. For instance, we may define the simplicity level of a game in terms of the smallest (in an inclusion sense) set of future histories that an agent must see as simple in the sense of Section 4 in order to play the equilibrium strategy correctly, or as the highest k that still allows the agent to play k-simple strategies correctly. We may similarly define the measure of sophistication of experimental subjects as the highest k that allows the subjects to play k-simple strategies correctly.

In sum, the sophistication of agents may vary across applications, and so it is important to have a range of simplicity standards. For sophisticated agents, a weaker simplicity standard ensures they play the intended strategies, allowing the designer more flexibility on other objectives; however, for less sophisticated agents, a stronger standard of simplicity may need to be imposed to ensure the intended strategies are played, with potential limitations on flexibility. Understanding the simplicity of games and the simplicity-flexibility trade-off requires an adaptable approach to thinking about simplicity. This paper puts forth one such proposal, though there is much work still to be done in fully exploring this trade-off and testing various simplicity standards empirically.

### APPENDIX A: PROOFS

This appendix contains the central elements of the proofs of our main theorems. All lemmas used in these proofs, as well as Theorem 6 and Lemma 1 from the main text, are proven in the Supplemental Material.

# A.1. Pruning Principle

Given a game  $\Gamma$  and strategy profile  $(S_i(\succ_i))_{i\in\mathcal{N}}$ , the *pruning* of  $\Gamma$  with respect to  $(S_i(\succ_i))_{i\in\mathcal{N}}$  is a game  $\Gamma'$  that is defined by starting with  $\Gamma$  and deleting all histories of  $\Gamma$  that are never reached for any type profile. Li (2017b) introduced the following *pruning principle*: if  $(S_i(\succ_i))_{i\in\mathcal{N}}$  is obviously dominant for  $\Gamma$ , then the restriction of  $(S_i(\succ_i))_{i\in\mathcal{N}}$  to  $\Gamma'$  is obviously dominant for  $\Gamma'$ , and both games result in the same outcome. Thus, for

<sup>&</sup>lt;sup>36</sup>A different approach to the trade-off between simplicity and flexibility was proposed by Li and Dworczak (2020), who studied strategy-proofness, obvious strategy-proofness, and strong obvious strategy-proofness. While we evaluate this trade-off for designers who never confuse the mechanism participants, they evaluated it for designers who can confuse participants. See also work in progress by Catonini and Xue (2021), who studied a weakening of one-step simplicity.

<sup>&</sup>lt;sup>37</sup>For a test of the first claim, see also Breitmoser and Schweighofer-Kodritsch (2019).

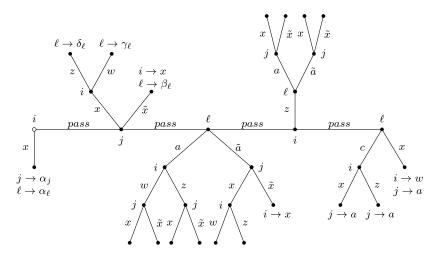


FIGURE 4.—A game in which greedy strategies are two-step simple and for which no equivalent one-step simple mechanism exists.

any OSP mechanism, we can find an equivalent OSP pruned mechanism. For strong obvious dominance, the pruning principle remains valid: if  $(S_i(\succ_i))_{i\in\mathcal{N}}$  is strongly obviously dominant for  $\Gamma$ , then the restriction of  $(S_i(\succ_i))_{i\in\mathcal{N}}$  to its pruning  $\Gamma'$  is strongly obviously dominant for  $\Gamma'$ , and both games result in the same outcome.

## A.2. Proof of Theorem 2

In light of Corollary 1, it is sufficient to prove the result for  $k < \infty$  and k' = k + 1. For k = 0, the result follows from Theorems 6 and 8, applied to a single-unit auction with transfers. Theorem 6 shows that in such a setting, personal clock auctions are efficient and OSS, while Theorem 8 implies that an efficient, SOSP (k = 0) mechanism does not exist when there are at least two agents whose valuations are drawn i.i.d. from at least three values (see also footnote 35). For k = 1, we construct below a 2-step simple social choice rule that cannot be one-step implemented; we conclude the proof by extending this example to any larger k.

Consider an object allocation environment without transfers in which agents demand exactly one object each. There are at least three agents  $i,j,\ell$  and the objects included in the game  $\Gamma$  are shown in Figure 4. Each branch of the game tree represents a clinching action where the agent clinches the labeled object  $(x,\tilde{x},\text{etc.})$ . The notation such as " $\ell \to \gamma$ " below terminal nodes denotes that agent  $\ell$  is assigned to object  $\gamma$  at this node, without needing to take any action. The root of the game is agent i's choice between clinching x and passing. If i clinches x at the first move, then the game immediately ends with j assigned  $\alpha_j$  and  $\ell$  assigned  $\alpha_\ell$ , and further, this is the only terminal history at which j receives  $\alpha_j$  and  $\ell$  receives  $\alpha_\ell$ . Similarly, there are objects  $\beta_\ell$ ,  $\gamma_\ell$ , and  $\delta_\ell$  that agent  $\ell$  receives only at the denoted terminal histories, and nowhere else in the game.

It is straightforward to check that  $(\Gamma, S_{\mathcal{N}, \mathcal{H}})$ , where  $S_{\mathcal{N}, \mathcal{H}}$  is a profile of greedy strategic collections, is k-step implementable for any  $k \geq 2$ ; in particular, this implies that  $\Gamma$  is OSP. It is also easy to check that  $\Gamma$  itself is not OSS: the type of i that ranks  $w \succ_i x \succ_i z$  has no one-step simple strategic plan when choosing between x and passing at the first move of the game. Showing that the social choice rule implemented by  $(\Gamma, S_{\mathcal{N}, \mathcal{H}})$  cannot

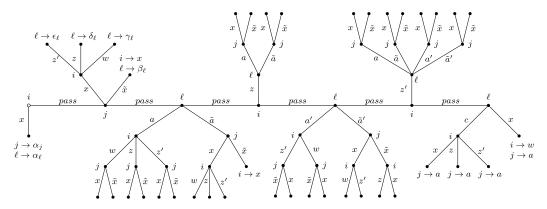


FIGURE 5.—A game in which greedy strategies are three-step simple and for which no equivalent two-step simple mechanism exists.

be OSS-implemented by any other mechanism is subtler, and we relegate the proof of the following lemma establishing this statement to Appendix B.1 of the Supplemental Material.

## LEMMA A.1: No one-step simple mechanism is equivalent to $(\Gamma, S_{\mathcal{N}, \mathcal{H}})$ .

For k=2, game  $\Gamma^{(2)}$  in Figure 5 is an example that is k'-step simple for any k'>k, but for which no equivalent k-step simple mechanism exists. This game is similar in structure to that of Figure 4, but has the following additions:

- (i) In the subgame following i passing and j clinching x at its first move, we add the possibility of i clinching z'. In this way, we assure that i can then clinch any possible and not previously clinchable object.<sup>38</sup>
- (ii) In the subgame following i and j passing and  $\ell$  clinching a at its first move, we add the possibility of i clinching z' (following which j can clinch x and  $\tilde{x}$ ). In this way, we assure that i can clinch any possible and not previously clinchable object.
- (iii) Following i's pass at its second move on the focal path, we add a node at which  $\ell$  can clinch two new objects a' and  $\tilde{a}'$  (following the clinching of a', agent i can clinch any possible not previously clinchable object, and then j can clinch any previously clinchable object; following the clinching of  $\tilde{a}'$ , agent j can clinch any previously clinchable object, and then following the clinching of x, agent i can clinch any possible but not previously clinchable objects, while following the clinching of  $\tilde{x}$ , agent i can clinch any previously clinchable object).
- (iv) Following the pass at the added node for  $\ell$ , we add a node at which i can clinch an additional object z'. Following i clinching z',  $\ell$  and then j can clinch any object they could clinch previously.

To prove the theorem for arbitrary  $k \geq 2$ , we recursively create game  $\Gamma^{(k)}$  by adding to game  $\Gamma^{(k-1)}$  further objects  $z^{(k)}$ ,  $a^{(k)}$ , and  $\tilde{a}^{(k)}$ , and then adding the analogues of subgames (i)–(iv). In the analogues of subgames (i)–(ii), we now allow i to additionally clinch  $z^{(k)}$ ; in the analogue of (iii),  $a^{(k)}$  and  $\tilde{a}^{(k)}$  play the roles of a and  $\tilde{a}$ , and in the analogue of (iv),  $z^{(k)}$  plays the role of z.

<sup>&</sup>lt;sup>38</sup>This property and the property in (ii) were also true in game  $\Gamma$  in Figure 4 and these two modifications simply reestablish these properties for the game  $\Gamma^{(2)}$  in Figure 5, in which z' becomes possible for i.

It is straightforward to check that  $(\Gamma^{(k)}, S_{\mathcal{N}, \mathcal{H}}^{(k)})$  is (k+1)-step simple but not k-step simple, where  $S_{\mathcal{N}, \mathcal{H}}^{(k)}$  is a profile of greedy strategic collections. Showing that no equivalent mechanism is k-step simple is done similarly to the k=1 case. The details can be found in Appendix B.1 of the Supplemental Material.

LEMMA A.2: For any  $k \geq 2$ , no k-step-simple mechanism is equivalent to  $(\Gamma^{(k)}, S_{\mathcal{N}, \mathcal{H}}^{(k)})$ .

Lemmas A.1 and A.2 establish the result for  $k \ge 1$ .

## A.3. Proof of Theorem 3

The proof develops the proof of the similar result for OSP in Li (2017b). For one direction of implication, suppose the strategic plan  $S_{i,I^*}$  is simply dominant from the perspective of  $I^* \in \mathcal{I}_i$  in  $\Gamma$ . Then any outcome that is possible after playing  $S_{i,I^*}$  at all information nodes  $I \in \mathcal{I}_{i,I^*}$  is weakly better than any outcome that is possible after playing  $S_i'(I^*) \neq S_{i,I^*}(I^*)$  in  $\Gamma$ , and hence the analogue of this "weakly better" comparison applies to the counterparts of these actions in any game  $\Gamma'$  that is indistinguishable from  $\Gamma$  from the perspective of i at  $I^*$  (by condition (3) of indistinguishability). Hence, in any such  $\Gamma'$ , every strategy  $S_i'$  that calls for playing the counterparts of actions  $S_{i,I^*}(I)$  for counterparts of all  $I \in \mathcal{I}_{i,I^*}$  weakly dominates any strategy  $S_i''$  that does not call for playing the counterpart of  $S_{i,I^*}(I^*)$  at the counterpart of  $I^*$ .

For the other direction of implication, fix information set  $I^*$  at which i moves, preference ranking  $\succeq_i$  of agent i, and a partial strategic plan  $S_{i,I^*}$  such that in every game  $\Gamma'$  that is indistinguishable from  $\Gamma$  from the perspective of agent i at  $I^*$ , any strategy  $S_i'$  that plays counterparts of  $S_{i,I^*}(I)$  for all counterparts of  $I \in \mathcal{I}_{i,I^*}$  weakly dominates any strategy  $S_i''$  that plays at the counterpart of  $I^*$  another action than the counterpart of  $S_{i,I^*}(I^*)$ . Our goal is to show that any outcome that is possible when i follows  $S_{i,I^*}$  at information sets  $\mathcal{I}_{i,I^*}$  is  $\succeq_i$ -weakly preferred to any outcome that is possible after i plays any  $a \neq S_{i,I^*}(I^*)$  at  $I^*$  in game  $\Gamma$ . To prove it, consider  $\Gamma'$  which differs from  $\Gamma$  only in that all moves of agent i and other agents that follow history i but are not in  $\mathcal{I}_{i,I^*}$  are made by Nature instead of the party making them in  $\Gamma$  and that Nature puts positive probability on all its possible moves. Notice that such  $\Gamma'$  is indistinguishable from  $\Gamma$  from the perspective of i at  $I^*$ . As in  $\Gamma'$  any strategy that selects counterparts of  $S_{i,I^*}$  at any counterpart of  $I \in \mathcal{I}_{i,I^*}$  weakly dominates any strategy  $S_i''$  that selects a at the counterpart of  $I^*$ , we conclude from condition (3) of indistinguishability that, in  $\Gamma$ , any outcome that is possible after i follows  $S_{i,I^*}$  at information sets in  $\mathcal{I}_{i,I^*}$  is weakly better than any outcome that is possible following a.

## A.4. Proof of Theorem 5

Section 4 introduces the notions of possible and clinchable payoffs at a history h, and the sets of such payoffs, denoted  $P_i(h)$  and  $C_i(h)$ , respectively. For the proof, we also need the notion of a guaranteeable payoff: a payoff x is guaranteeable for i at h if there is some continuation strategy  $S_i$  such that i receives payoff x at all terminal histories  $\bar{h} \supseteq h$  that are consistent with i following  $S_i$ . We use  $G_i(h)$  to denote the set of payoffs that are guaranteeable for i at history h.

The proof is broken down into five steps, stated as Lemmas A.3–A.7 below. The proofs of these lemmas can be found in Appendix B.2 of the Supplemental Material. First, we check there that all millipede games with greedy strategies are OSP, establishing one direction of the theorem.

LEMMA A.3: Millipede games with greedy strategies are obviously strategy-proof.

Given Li's pruning principle (see Section A.1), the converse implication of Theorem 5—that all OSP mechanisms are equivalent to millipedes—follows from the remaining four lemmas.<sup>39</sup> Lemma A.4 develops Theorem 4 (see this theorem for a discussion):

LEMMA A.4: Every OSP game is equivalent to an OSP game with perfect information in which Nature moves at most once, as the first mover.

Lemma A.5 shows that if a game is OSP, then at every history, for all actions a with the exception of possibly one special action  $a^*$ , all payoffs that are possible following a are also guaranteeable at h.<sup>40</sup>

LEMMA A.5: Let  $\Gamma$  be an obviously strategy-proof game of perfect information that is pruned with respect to the obviously dominant strategy profile  $(S_i(\succ_i))_{i\in\mathcal{N}}$ . Consider a history h where agent  $i_h = i$  is called to move. There is at most one action  $a^* \in A(h)$  such that  $P_i((h, a^*)) \nsubseteq G_i(h)$ .

The above lemma leaves open the possibility that there are several actions that can ultimately lead to multiple final payoffs for i, which can happen when different payoffs are guaranteeable for i by following different strategies in the future of the game. The next lemma shows that if this is the case, we can always construct an equivalent OSP game such that all actions except for possibly one are clinching actions.

LEMMA A.6: For any OSP game  $\Gamma$ , there exists an equivalent OSP game  $\Gamma'$  such that the following hold at each  $h \in \mathcal{H}$  (where i is the agent called to move at h):

- (i) At least |A(h)| 1 actions at h are clinching actions.
- (ii) For every payoff  $x \in G_i(h)$ , there exists an action  $a_x \in A(h)$  that clinches x for i.
- (iii) If  $P_i(h) = G_i(h)$ , then all  $a \in A(h)$  are clinching actions and  $i_{h'} \neq i$  for any  $h' \supseteq h$ .

The final lemma of the proof establishes the payoff guarantees in the game constructed in the previous lemmas.

LEMMA A.7: Let  $(\Gamma, S_N)$  be an obviously strategy-proof mechanism that satisfies the conclusions of Lemmas A.4 and A.6. At all h, if there exists a previously unclinchable payoff z that becomes impossible for agent  $i_h$  at h, then  $C_{i_h}^{\subset}(h) \subseteq C_i(h)$ .

This lemma concludes the proof of Theorem 5.

# A.5. Proof of Theorem 7

We first prove the second statement. Let  $\Gamma$  be a monotonic millipede game. Fix an agent i, and, for any history  $h^*$  at which i moves, let  $\bar{x}_{h^*} = \text{Top}(\succ_i, P_i(h^*))$  and  $\bar{y}_{h^*} = \text{Top}(\succ_i, C_i(h^*))$ . Let  $\mathcal{H}_{i,h^*} = \{h \in \mathcal{H}_i | h^* \subsetneq h' \subsetneq h \implies h' \notin \mathcal{H}_i\}$  be the set of one-step simple nodes. Consider the following strategic plan for any  $h^*$ :

<sup>&</sup>lt;sup>39</sup>We actually prove a slightly stronger statement, which is that every OSP game is equivalent to a millipede game that satisfies the following additional property: for all i, all h at which i moves, and all  $x \in G_i(h)$ , there exists an action  $a_x \in A(h)$  that clinches x (see Lemma A.6 below).

<sup>&</sup>lt;sup>40</sup>We emphasize the distinction between a payoff x being "guaranteeable" versus "clinchable": the latter means the agent receives x at all terminal histories, while the former means there is a continuation strategy  $S_i$  such that she receives x at all terminal histories consistent with  $S_i$ .

- If  $\bar{x}_{h^*} \in C_i(h^*)$ , then  $S_{i,h^*}(h^*) = a_{\bar{x}_{h^*}}$ , where  $a_{\bar{x}_{h^*}} \in A(h^*)$  is a clinching action for  $\bar{x}_{h^*}$ .
- If  $\bar{x}_{h^*} \notin C_i(h^*)$ , then  $S_{i,h^*}(h^*) = a^*$  (*i* passes at  $h^*$ ), and, for any other  $h \in \mathcal{H}_{i,h^*}$ :
  - If  $P_i(h^*) \setminus C_i(h^*) \subseteq C_i(h)$ , then  $S_{i,h^*}(h^*) = a_{\bar{x}_{h^*}}$ .
- Else, we have  $C_i(h^*) \subseteq C_i(h)$  (by monotonicity) and we set  $S_{i,h^*}(h^*) = a_{\bar{y}_{h^*}}$ . It is straightforward to verify that this strategic plan is one-step dominant at any  $h^*$ , and thus the corresponding strategic collection  $(S_{i,h^*})_{h^* \in \mathcal{H}_i}$  is also one-step dominant.

In order to prove the first statement, let  $(\Gamma, S_{\mathcal{N}, \mathcal{H}})$  be a millipede mechanism with a profile of one-step dominant strategic collections  $S_{\mathcal{N}, \mathcal{H}}$ . Begin by constructing an equivalent millipede mechanism that satisfies Lemma A.6. Note that the transformations used in the proof to construct the equivalent millipede mechanism are one-step dominance preserving—that is, if  $(\Gamma, S_{\mathcal{N}, \mathcal{H}})$  was an OSS millipede mechanism before the transformation, then the transformed game  $(\Gamma', S'_{\mathcal{N}, \mathcal{H}})$  is another OSS millipede mechanism that satisfies Lemma A.6. It remains to show the following:

LEMMA A.8: Any OSS millipede mechanism that, at each  $h \in \mathcal{H}$ , satisfies conditions (i), (ii), and (iii) of Lemma A.6 is monotonic.

We prove this lemma in Appendix B.4 of the Supplemental Material.

## A.6. Proof of Theorem 8

That sequential choice mechanisms are SOSP is immediate from the definition, and so we focus on proving that every SOSP mechanism is equivalent to a sequential choice mechanism. Following the same reasoning as in the proof of Lemma A.4, given any SOSP mechanism, we can construct an equivalent SOSP mechanism of perfect information in which Nature moves at most once, as the first mover. It remains to analyze the subgame after a potential move by Nature and to show that every perfect-information SOSP mechanism in which there are no moves by Nature is equivalent to a sequential choice mechanism.

Let  $(\Gamma, S_N)$  be such a mechanism. In line with the discussion in Section A.1, we can assume that  $\Gamma$  is pruned. By Lemma 1, each agent i can have at most one payoff-relevant history along any path of  $\Gamma$ , and this history (if it exists) is the first time i is called to play. Consider any such history  $h_0^i$ . If there is some other history  $h' \supset h_0^i$  at which i is called to play, then history h' must be payoff-irrelevant for i; in other words, there is some payoff x such that  $P_i((h', a')) = \{x\}$  for all  $a' \in A(h')$ . Using the same technique as in the proof of Lemma A.6, we construct an equivalent pruned game in which at history  $h_0^i$ , i is asked to also choose her actions for all successor histories  $h' \supset h_0^i$  at which she might be called to play, and then is not called to play again after  $h_0^i$ . Since all of these future histories were payoff-irrelevant for i, the new game continues to be strongly obvious dominant for i. Strong obvious dominance is also preserved for all  $j \neq i$ , since having i make all of her choices earlier only shrinks the set of possible outcomes any time j is called to move, and thus, if some action was strongly obviously dominant in the old game, the analogous action(s) will be strongly obviously dominant in the new game. Repeating this for every agent and every history, we construct a pruned SOSP game  $\Gamma'$  that is equivalent to  $\Gamma$  and in which each agent is called to move at most once along any path of play. It remains to show the following:

LEMMA A.9:  $\Gamma'$  with greedy strategies is a sequential choice mechanism.

We prove this lemma in Appendix B.6 of the Supplemental Material.

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